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Council of the Great City Schools

parent ROADMAP

SUPPORTING YOUR CHILD IN HIGH SCHOOL
MATHEMATICS

America's schools are working to provide higher quality instruction than ever before.

The way we taught students in the past simply does not prepare them for the higher demands of college and careers today and in the future. Your school and schools throughout the country are working to improve teaching and learning to ensure that all students will graduate high school with the skills they need to be successful.

In high school mathematics, the emphasis in instruction is on a few interrelated big ideas that connect the study of mathematics to science, technology, and engineering. Teachers will concentrate on teaching these big ideas using complex and challenging math content, enabling students to illustrate their thinking and apply their knowledge in many different ways. Students will learn how to use mathematics to analyze and respond to real-world issues and challenges, as they will be expected to do in college and the workplace.

What your child will be learning in high school mathematics

In high school, students will develop a deep understanding of mathematical concepts and use mathematical ways of thinking to solve real-world problems. Unlike previous grades where learning objectives are organized by grade level, high school learning objectives are organized by concepts—such as algebra, functions, or geometry—that students will learn and master in various mathematics courses. These concepts build on what students learned in grade eight and move toward greater depth of knowledge and skills throughout high school. Here's a brief snapshot of some of the work students will be doing in these areas:

- Creating and solving *equations* (mathematical statements that use letters to represent unknown numbers, such as 2x-6y+z=14) with two or more variables to describe numbers or relationships
- Building an understanding of *rational numbers* (such as ¾) to include *rational expressions* (such as 3/(x-4))
- Using the structure of an *expression* to identify ways to rewrite it. For example, recognizing that x^8-y^8 is the difference between two squares and can also be written $(x^4)^2$-$(y^4)^2$
- Adding, subtracting, and multiplying *polynomials* (an expression with multiple terms such as $5xy^2+2xy-7$)
- Interpreting the *slope* of a line as the rate of change in two variables and the *intercept* as the constant term in a *linear model*
- Building and analyzing *functions* that describe relationships between quantities and using *function notation* (for example, f(x) denotes the output of f corresponding to the input of x)
- Representing and performing operations with *complex numbers* (numbers such as 3+5*i*, where *i* is an imaginary number and $i=\sqrt{-1}$)
- Understanding the rules of *probability* and using them to interpret data and evaluate the outcomes of decisions
- Distinguishing between correlation and causation
- Interpreting quantitative and categorical data
- Understanding and proving geometric *theorems* (mathematical statements whose truth can be proven on the basis of previously proven or accepted statements)
- Using algebraic reasoning to prove geometric theorems
- Applying geometric concepts to model real-life situations

Complex numbers are used in many scientific fields, including engineering and quantum physics.

Here are just a few examples of how students will develop and apply an understanding of structure and patterns in algebraic expressions.

Grade Eight Mathematics	High School Mathematics	
• Understand the connections between proportional relationships, lines, and linear equations • Use linear equations to graph proportional relationships, interpreting the unit rate as the slope of a line • Know and apply the properties of integer exponents (positive numbers, negative numbers, or 0) to write equivalent expressions (such as $4^2 \bullet 4^3 = 4^5$) "•" is a multiplication symbol students use in grade eight and beyond	• Interpret the structure of an expression • Use the structure of an expression to identify ways to rewrite it. For example, $x^4-y^4=(x^2)^2-(y^2)^2$ • Interpret one or more parts of an expression individually. For example, interpret $6+(x-2)^2$ as the sum of a constant and the square of x-2 • Solve quadratic equations (equations that include the square of a variable, such as $5x^2-3x+3=0$) • Factor a quadratic expression to reveal the zeros of the function it defines	• Write expressions in equivalent forms to solve problems • Use the properties of exponents to transform and evaluate expressions. For example, interpret $(8^{2/3})^2=(8^{1/3})^4=2^4=16$ • Derive the formula for the sum of a finite geometric series, and use the formula to solve problems. For example, 3, 12, 48, 192 is a finite series where the ratio between each term is 4; 12/3=48/12=192/48=4 • Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines

In grade eight, students solved real-world problems involving rates and discount, focusing on the computation needed to determine the final price. In high school, the emphasis is not about merely computing the final price, but rather using the structure of the answer to make a general argument.

Sample Problem

Judy is working at a retail store over summer break. A customer buys a \$50 shirt that is on sale for 20% off. Judy computes the discount, then adds sales tax of 10%, and tells the customer how much he owes. The customer insists that Judy first add the sales tax and then apply the discount. He is convinced that this way he will save more money because the discount amount will be larger.

a. Is the customer right?

b. Does the answer to part (a) depend on the numbers used or would it work for any percentage discount and any sales tax percentage? Find a convincing argument using algebraic expressions and/or diagrams for this more general scenario.

Solution:

Judy's solution:

After the 20% discount, the shirt will cost 80% of the original price. 0.80(\$50)=\$40

The tax will be 10% of this reduced price. 0.10(\$40)=\$4

The final cost will be the reduced price plus tax. \$40+\$4=\$44

The equation for finding this answer is \$50(0.80)(1.10)=\$44

Customer's solution:

Before the 20% discount, the shirt cost \$50. The tax will be 10% of this price. 0.10(\$50)=\$5

The cost before the discount would be price plus tax. \$50+\$5=\$55

After the 20% discount, the shirt will cost 80% of this price. 0.80(\$55)=\$44

The equation for finding this answer is \$50(1.10)(0.80)=\$44

In this problem, students apply their understanding that changing the order of quantities in a multiplication problem doesn't matter (known as the commutative property of multiplication). Students also show that given the structure of the equation used to find the answers, the answer would apply to any given combination of price, discount, and tax. For example, if we let P represent the original price, s represent the sale percentage, and t represent the tax percentage, students see that they can generalize the results.

Judy: $P\,(1-s/100)(1+t/100)$

Customer: $P\,(1+t/100)(1-s/100)$

Task adapted from www.illustrativemathematics.org.

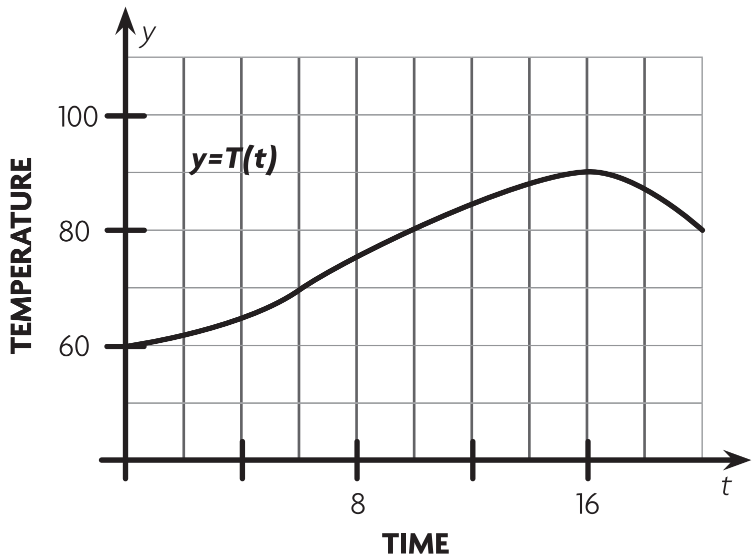
Here are just a few examples of how students connect functions, algebra, and modeling to describe relationships between quantities.

Grade Eight Mathematics	High School Mathematics	
• Understand that a function is a rule that assigns to each input exactly one output, and the graph of a function is a set of ordered pairs consisting of an input and a corresponding output • Compare the properties of two functions each represented in a different way (for example, in a table, graph, equation, or description) • Determine the rate of change and initial value of a function based on a description of a proportional relationship or of at least two given (x,y) values	• Understand and use function notation (for example, f(x) denotes the output of f corresponding to the input of x) • Interpret functions in terms of the context • Calculate and interpret the average rate of change of a function presented in a graph or table over a given interval • Graph functions symbolically and show key features of the graph, by hand or using technology (such as graphing calculators and computer programs) for more complicated cases	• Write a function defined by an expression in different but equivalent forms • For a function that models a relationship between two quantities, interpret key features of graphs and tables, including intercepts, intervals where the function is increasing or decreasing, relative maximums and minimums, etc. • Construct, compare, and apply linear, quadratic, and exponential models to solve problems

To promote fluency with functions representing proportional relationships, students begin by interpreting function notation in context. For example, if h is a function that relates Shea's height in inches to her age in years, then h(8)=50 means, "When Shea is eight years old, she is 50 inches tall."

Sample Problem

The figure shows the graph of T, the temperature (in degrees Fahrenheit) over one particular 20-hour period as a function of time t.



a. Estimate $T(14)$.

b. If $t=0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.

c. From the graph, estimate the highest temperature during this 20-hour period.

d. If Anya wants to go for a two-hour hike and return before the temperature is over 80 degrees, when should she leave?

Solution:

In this task, $T(14)$ means that 14 hours after midnight, the temperature is a little less than 90 degrees Fahrenheit; $T(14)$ is 2:00 p.m. The highest temperature on the graph is about 90 degrees. The temperature was decreasing between 4:00 p.m. and 8:00 p.m. It might have continued to decrease after that, but there is no information about the temperature after 8:00 p.m. If Anya wants to go for a two-hour hike and return before the temperature is over 80 degrees, then she should start her hike before 8:00 a.m.

Note: This is a straight-forward assessment task of reading and interpreting graphs. It requires an understanding of function notation and reinforces the idea that when a variable represents time, $t = 0$ is chosen as an arbitrary point in time and positive times are interpreted as times that happen after that point.

Task adapted from www.illustrativemathematics.org.

Helping your child learn outside of school

1. Show your enthusiasm for your child's study of mathematics.
2. Encourage your child to be persistent; make sure that he or she knows that mathematics requires patience, practice, and time to think and reflect.
3. Urge your child to ask the teacher questions either during or after class.
4. Encourage your child to review class notes every night. If there is something he or she doesn't understand, tell your child to look at the answers and work backwards to determine how the solution was found.

Partnering with your child's teacher

Don't be afraid to reach out to your child's teacher—you are still an important part of your child's education. Ask to see samples of your child's work and discuss his or her progress with the teacher using questions like:

- Where is my child excelling? How can I support this success?
- What do you think is giving my child the most trouble? How can I help my child improve in this area?
- Are there options provided by the school for enrichment experiences in mathematics, science, technology, or engineering (STEM areas) or for students having difficulty in mathematics?
- Is there a homework hotline or some other resource outside of class for students to ask questions about their homework or what they are learning?

Additional Resources

For more information on the Common Core State Standards for mathematics, go to http://www.corestandards.org/about-the-standards/key-points-in-mathematics or http://www.commoncoreworks.org.

For more information about the progression of mathematical ideas go to www.illustrativemathematics.org.

For more information on Montana's Content Standards, visit http://opi.mt.gov/Educators/Teaching-Learning/K-12-Content-Standards-Revision.